42

NUCLEAR PHYSICS

Conceptual Questions

42.1. **(a)** ^{16}O and ^{18}O $(Z=8)$; and ^{18}Ne and ^{20}Ne $(Z=10)$. **(b)** ^{18}O, ^{18}F, ^{18}Ne (all have $A=18$). **(c)** ^{16}O and ^{18}O; and ^{18}Ne and ^{20}Ne; different isotopes of the same element are chemically nearly identical. **(d)** ^{18}O and ^{20}Ne; and ^{16}O and ^{18}Ne.

42.2. **(a)** More than, because there are so many more nucleons. **(b)** Less tightly because for $A=200$, $B/A=8$ Mev/nucleon, but for $A=60$, $B/A=8.8$ Mev/nucleon.

42.3. **(a)** There must be an attractive force to hold the nucleus together that will overcome the electrostatic repulsive force between the protons. **(b)** There is no evidence for nuclear forces outside the nucleus.

42.4. **(a)** Impossible, only two neutrons are allowed in the $n=1$ state. **(b)** Impossible, the pairs of protons and neutrons must have one spin up and one spin down. **(c)** Excited nuclear state with one neutron excited to the $n=3$ level.

42.5. **(a)** No, the alpha particle is added to the wrong nucleus. **(b)** No, the original nucleus would need to lose four nucleons, not two. **(c)** Yes. **(d)** No, in beta decay A does not change.

42.6. $N=N_0e^{-t/\tau}\Rightarrow t=-\tau\ln\left(\dfrac{N}{N_0}\right)=-(10\text{ s})\ln\left(\dfrac{250}{1000}\right)=13.86\text{ s}\approx 14\text{ s}$

42.7. **(a)** Gamma decay can lower the excited proton to the $n=2$ level. **(b)** Beta-minus decay can change a neutron to a proton. **(c)** No decay is allowed.

42.8. The nuclear radiation would ionize some of the matter and break molecular bonds in apple A. This could drive some chemical reactions that would not occur in apple B. Apple A would not become radioactive.

42.9. ^{137}Cs would be most useful because after the beta decay a gamma ray is emitted. Alphas and betas do not penetrate material well, and so would not come out of the body to be detected, but the gamma ray penetrates material well and so could escape the body and be detected.

Exercises and Problems

Exercises

Section 42.1 Nuclear Structure

42.1. Model: The nucleus is composed of Z protons and $A-Z$ neutrons.

Solve: (a) ^{6}Li has $Z=3$ protons and $3-3=3$ neutrons.

(b) ^{54}Cr has $Z=24$ protons and $54-24=30$ neutrons.

(c) ^{54}Fe has $Z=26$ protons and $54-26=28$ neutrons.

(d) ^{220}Rn has $Z=86$ protons and $220-86=134$ neutrons.

42.2. Model: The nucleus is composed of Z protons and $A-Z$ neutrons.

Solve: (a) ^{3}He has $Z=2$ protons and $3-2=1$ neutron.

(b) ^{32}P has $Z=15$ protons and $32-15=17$ neutrons.

(c) ^{32}S has $Z=16$ protons and $32-16=16$ neutrons.

(d) ^{238}U has $Z=92$ protons and $238-92=146$ neutrons.

42.3. Solve: (a) The radius and diameter of the nucleus of ^{4}He are

$$r=r_0A^{1/3}=(1.2\text{ fm})(4)^{1/3}=1.90\text{ fm}\Rightarrow d=3.8\text{ fm}$$

(b) For ^{56}Fe, $r=(1.2\text{ fm})(56)^{1/3}=4.59\text{ fm}$ and $d=9.2\text{ fm}$.

(c) For ^{238}U, $r=(1.2\text{ fm})(238)^{1/3}=7.44\text{ fm}$ and $d=14.9\text{ fm}$.

42.4. Visualize: The masses of the nuclei are found by subtracting the mass of Z electrons from the atomic masses in Appendix C, and then converting to kg.

$$m(^{6}\text{Li})=6.015121\text{ u}-3(0.000548)\text{ u}=6.0135\text{ u}$$

$$m(^{207}\text{Pb})=206.975871\text{ u}-82(0.000548)\text{ u}=206.9309\text{ u}$$

The radii are computed from $r=r_0A^{1/3}$ where $r_0=1.2$ fm. The densities are computed from

$$\rho=\frac{m}{\frac{4}{3}\pi r^3}$$

Solve: (a) For ^{6}Li:

$$m=6.0135\text{ u}\left(\frac{1.661\times10^{-27}\text{ kg}}{1\text{ u}}\right)=9.988\times10^{-27}\text{ kg}$$

$$r=r_0A^{1/3}=(1.2\times10^{-15}\text{ m})(6)^{1/3}=2.18\times10^{-15}\text{ m}\approx2.2\times10^{-15}\text{ m}$$

$$\rho=\frac{m}{\frac{4}{3}\pi r^3}=\frac{9.988\times10^{-27}\text{ kg}}{\frac{4}{3}\pi(2.18\times10^{-15}\text{ m})^3}=2.3\times10^{17}\text{ kg/m}^3$$

(b) For ^{207}Pb:

$$m=206.9309\text{ u}\left(\frac{1.661\times10^{-27}\text{ kg}}{1\text{ u}}\right)=3.437\times10^{-25}\text{ kg}$$

$$r=r_0A^{1/3}=(1.2\times10^{-15}\text{ m})(207)^{1/3}=7.10\times10^{-15}\text{ m}\approx7.1\times10^{-15}\text{ m}$$

$$\rho=\frac{m}{\frac{4}{3}\pi r^3}=\frac{3.437\times10^{-25}\text{ kg}}{\frac{4}{3}\pi(7.10\times10^{-15}\text{ m})^3}=2.3\times10^{17}\text{ kg/m}^3$$

Assess: The densities are very similar because the nucleons are tightly packed in both cases. All nuclei have similar densities. This is also a typical density for a neutron star.

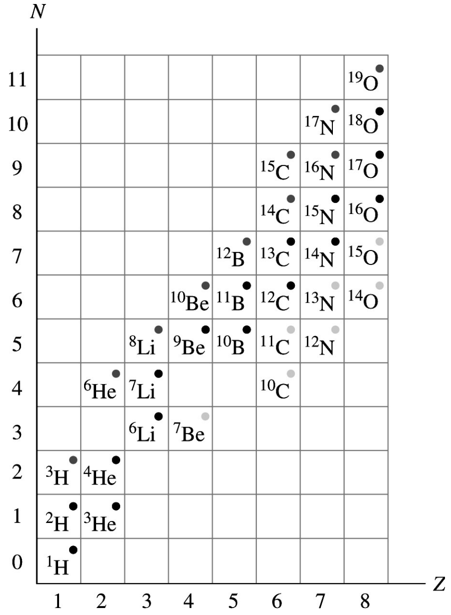
42.5. Solve: Using Equation 42.2,

$$r = r_0 A^{1/3} \Rightarrow 7.46 \text{ fm} = 2(1.2 \text{ fm})A^{1/3} \Rightarrow A = \left(\frac{7.46 \text{ fm}}{2.4 \text{ fm}}\right)^3 = 30$$

Only silicon has a stable isotope of $A = 30$.

Section 42.2 Nuclear Stability

42.6. Solve: The chart shows stable and unstable nuclei for all nuclei with $Z \le 8$. A black dot represents stable isotopes, a dark gray dot represents isotopes that undergo beta-minus decay, and a light gray dot represents isotopes that undergo beta-plus decay or electron-capture decay.



42.7. Solve: (a) $A = 36$, for which ^{36}S and ^{36}Ar are stable.

(b) Nuclei with $A = 8$ and $A = 5$ have no stable nuclei.

42.8. Solve: From Equation 42.6, the binding energy for ^{3}H is

$$B = (Zm_\text{H} + Nm_\text{n} - m_\text{atom}) = 1(1.00783 \text{ u}) + 2(1.00866 \text{ u}) - 3.01605 \text{ u}$$
$$= 0.00910 \text{ u} \times 931.49 \text{ MeV/u} = 8.48 \text{ MeV}$$

The binding energy per nucleon is $\frac{1}{3}(8.48 \text{ MeV}) = 2.83 \text{ MeV}$.

For ^{3}He, the binding energy is

$$B = Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 2(1.00783 \text{ u}) + 1(1.00866 \text{ u}) - 3.01603 \text{ u}$$
$$= 0.00829 \text{ u} \times 931.49 \text{ MeV/u} = 7.72 \text{ MeV}$$

The binding energy per nucleon is $\frac{1}{3}(7.72 \text{ MeV}) = 2.57 \text{ MeV}$.

42.9. Solve: From Equation 42.6, the binding energy for ^{129}I is

$$B = Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 53(1.00783 \text{ u}) + 76(1.00866 \text{ u}) - 128.904984 \text{ u}$$
$$= 1.168166 \text{ u} \times 931.49 \text{ MeV/u} = 1088.13495 \text{ MeV} \approx 1088.1 \text{ MeV}$$

The binding energy per nucleon is $\frac{1}{129}(1088.13495\ \text{MeV}) = 8.4352\ \text{MeV} \approx 8.44\ \text{MeV}$.

For ^{129}Xe, the binding energy is

$$\begin{aligned} B &= Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 54(1.00783\ \text{u}) + 75(1.00866\ \text{u}) - 128.904779\ \text{u} \\ &= 1.167541\ \text{u} \times 931.49\ \text{MeV/u} = 1087.55277\ \text{MeV} \approx 1087.6\ \text{MeV} \end{aligned}$$

The binding energy per nucleon is $\frac{1}{129}(1087.55277\ \text{MeV}) = 8.430641\ \text{MeV} \approx 8.43\ \text{MeV}$.

42.10. Solve: From Equation 42.6, the binding energy for ^3He is

$$\begin{aligned} B &= Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 2(1.00783\ \text{u}) + 1(1.00866\ \text{u}) - 3.016029\ \text{u} \\ &= 0.008291\ \text{u} \times 931.49\ \text{MeV/u} = 7.723\ \text{MeV} \end{aligned}$$

The binding energy per nucleon is $\frac{1}{3}(7.723\ \text{MeV}) = 2.57\ \text{MeV}$. For ^4He, the binding energy is

$$\begin{aligned} B &= Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 2(1.00783\ \text{u}) + 2(1.00866\ \text{u}) - 4.002602\ \text{u} \\ &= 0.03038\ \text{u} \times 931.49\ \text{MeV/u} = 28.30\ \text{MeV} \end{aligned}$$

The binding energy per nucleon is $\frac{1}{4}(28.30\ \text{MeV}) = 7.07\ \text{MeV}$. Because the binding energy per nucleon is more for ^4He than for ^3He, ^4He is more tightly bound.

42.11. Solve: From Equation 42.6, the binding energy for ^{14}O is

$$\begin{aligned} B &= Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 8(1.00783\ \text{u}) + 6(1.00866\ \text{u}) - 14.008595\ \text{u} \\ &= 0.106005\ \text{u} \times 931.49\ \text{MeV/u} = 98.7426\ \text{MeV} \end{aligned}$$

The binding energy per nucleon is $\frac{1}{14}(98.7426\ \text{MeV}) = 7.05\ \text{MeV}$.

For ^{16}O, the binding energy is

$$\begin{aligned} B &= Zm_\text{H} + Nm_\text{n} - m_\text{atom} = 8(1.00783\ \text{u}) + 8(1.00866\ \text{u}) - 15.994915\ \text{u} \\ &= 0.137005\ \text{u} \times 931.49\ \text{MeV/u} = 127.6188\ \text{MeV} \end{aligned}$$

The binding energy per nucleon is $\frac{1}{16}(127.6188\ \text{MeV}) = 7.98\ \text{MeV}$. Thus, ^{16}O is slightly more tightly bound.

42.12. Visualize: Use the data in Appendix C.

Solve: 92.23% of silicon is ^{28}Si, 4.67% of silicon is ^{29}Si, and the remaining 3.10% of silicon is ^{30}Si. The chemical atomic mass of silicon is

$$0.9223(27.976927\ \text{u}) + 0.0467(28.976495\ \text{u}) + 0.0310(29.973770\ \text{u}) = 28.085509\ \text{u} \approx 28.0855\ \text{u}$$

Assess: There is a ^{31}Si isotope, but its half-life is so short it contributes nothing to the overall chemical atomic mass. The percentages above add up to 100%.

Section 42.3 The Strong Force

42.13. Model: The force is the negative of the slope of a potential energy graph.

Visualize: Please refer to Figure 42.8.

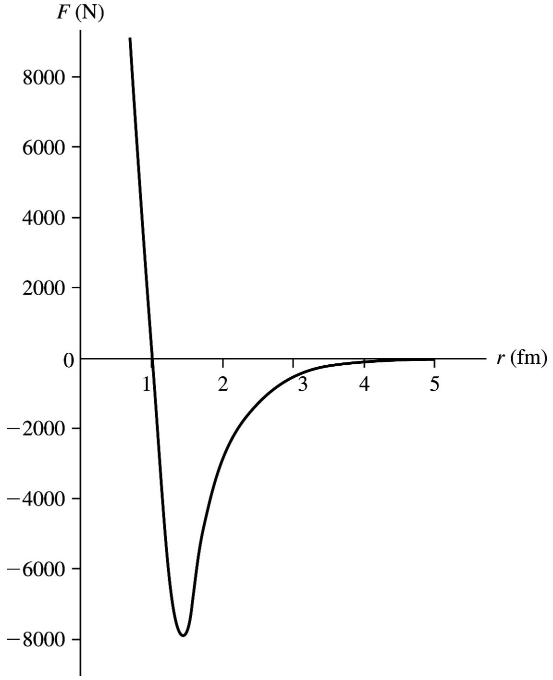
Solve: For the two nucleons that are separated by 1.5 fm, the potential energy is $U \approx -38\ \text{MeV}$. A line drawn tangent to the curve at this point goes from ≈ -70 MeV at 1 fm to ≈ -20 MeV at 2 fm. Its slope is

$$\frac{\Delta U}{\Delta r} \approx \frac{50\ \text{MeV}}{1.0\ \text{fm}} = \frac{50\ \text{MeV}}{1\times10^{-15}\ \text{m}} \times 1.6\times10^{-19}\ \text{J/eV} = 8000\ \text{J/m}$$

Thus $F = -\Delta U/\Delta r = -8000\ \text{N}$. The strength of the force is 8000 N.

42.14. Model: The force is the negative of the slope of a potential energy graph.

Solve: The force is zero when U is at its maximum negative value at $r \approx 1.0$ fm. The maximum force occurs at $r \approx 1.5$ fm where the slope is a maximum. The slope is approximately 8000 N, so $F = -8000$ N. For $r \le 1.0$ fm, the slope is negative and hence the force is positive. For $r \ge 1.0$ fm, the slope is positive and hence the force is negative.



42.15. Solve: From Figure 42.8, the nuclear potential energy at $r = 1.0$ fm is

$$U_{\text{nuclear}} = -50\text{ MeV} = -(50\text{ MeV})(1.6\times10^{-19}\text{ J/eV}) = -8.0\times10^{-12}\text{ J}$$

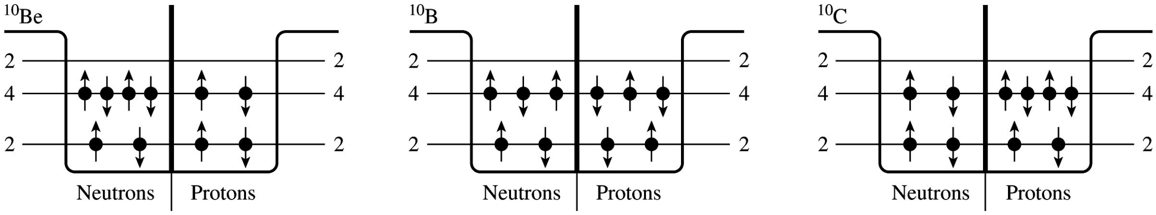
The gravitational potential energy is

$$U_{\text{grav}} = -\frac{Gm^2}{r} = -\frac{(6.67\times10^{-11}\text{ Nm}^2/\text{kg}^2)(1.67\times10^{-27}\text{ kg})^2}{1.0\times10^{-15}\text{ m}} = -1.86\times10^{-49}\text{ J}$$

$$\Rightarrow \frac{U_{\text{grav}}}{U_{\text{nuclear}}} = \frac{1.86\times10^{-49}\text{ J}}{8.0\times10^{-12}\text{ J}} = 2.3\times10^{-38}$$

Section 42.4 The Shell Model

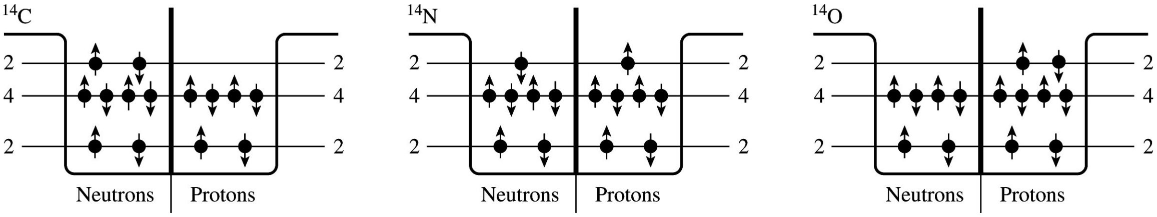
42.16. Solve: (a) The $A = 10$ nuclei listed in Appendix C are ^{10}Be, ^{10}B, and ^{10}C. Their nuclear energy level diagrams are shown in the figure below. ^{10}Be has $Z = 4$, so $N = 6$. ^{10}B has $Z = 5$, $N = 5$, and ^{10}C has $Z = 6$, $N = 4$. These three nuclei are called the $A = 10$ isobars.



(b) ^{10}B is a stable nucleus, but ^{10}Be and ^{10}C are radioactive. ^{10}Be undergoes beta-minus decay and ^{10}C undergoes beta-plus decay. In beta-minus decay, a neutron within the nucleus changes into a proton and an electron. In beta-plus decay, a proton changes into a neutron and a positron.

Assess: The above decays for ^{10}Be and ^{10}C are consistent with the fact that the line of stability follows the $N = Z$ line for $Z < 16$.

42.17. Solve: (a) The $A = 14$ nuclei listed in Appendix C are ^{14}C, ^{14}N, and ^{14}O. ^{14}C has $Z = 6$, so $N = 8$. ^{14}N has $Z = 7$, $N = 7$, and ^{14}O has $Z = 8$, $N = 6$. These 3 nuclei are the $A = 14$ isobars.



(b) ^{14}N is a stable nucleus, but ^{14}C and ^{14}O are radioactive. ^{14}C undergoes beta-minus decay and ^{14}O undergoes beta-plus decay. In beta-minus decay, a neutron within the nucleus changes into a proton and an electron. In beta-plus decay, a proton changes into a neutron and a positron.

Assess: The above decays for ^{14}C and ^{14}O are consistent with the fact that the line of stability follows the $N = Z$ line for $Z < 16$.

Section 42.5 Radiation and Radioactivity

42.18. Model: The number of radioactive atoms decreases exponentially with time.

Solve: The mass of remaining ^{131}Ba nuclei at time *t* is

$$N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

(a) After 1 day, $N = (250\ \mu\text{g})(1/2)^{1\ \text{day}/12\ \text{days}} = 236\ \mu\text{g}$.

(b) After 10 days, $N = (250\ \mu\text{g})(1/2)^{10\ \text{days}/12\ \text{days}} = 140\ \mu\text{g}$.

(c) After 100 days, $N = (250\ \mu\text{g})(1/2)^{100\ \text{days}/12\ \text{days}} = 0.775\ \mu\text{g}$.

Assess: Do not think that if half the nuclei decay during one half-life, then all will decay in two half-lives.

42.19. Model: The number of radioactive atoms decreases exponentially with time.

Solve: The number of remaining ^{226}Ra nuclei at time *t* is $N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$.

(a) After 200 years, $N = (1.0\times10^{10})(1/2)^{200\ \text{years}/1600\ \text{years}} = 9.17\times10^{9}$.

(b) After 2000 years, $N = (1.0\times10^{10})(1/2)^{2000\ \text{years}/1600\ \text{years}} = 4.20\times10^{9}$.

(c) After 20,000 years, $N = (1.0\times10^{10})(1/2)^{20{,}000\ \text{years}/1600\ \text{years}} = 1.73\times10^{6}$.

Assess: Do not think that if half the nuclei decay during one half-life, then all will decay in two half-lives.

42.20. Model: The number of atoms decays exponentially with time.

Solve: The number of remaining radioactive atoms at $t = 50$ min and $t = 200$ min is

$$N_{50} = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = (1.0\times10^{10})\left(\frac{1}{2}\right)^{(50\text{ min})/(100\text{ min})} = 7.07\times10^9\text{ atoms}$$

$$N_{200} = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = (1.0\times10^{10})\left(\frac{1}{2}\right)^{(200\text{ min})/(100\text{ min})} = 2.50\times10^9\text{ atoms}$$

Thus the number that decay between 50 min and 200 min is

$$N_{\text{decay}} = N_{50} - N_{200} = 4.6\times10^9\text{ atoms}$$

Each decay emits an alpha particle, so there are 4.6×10^9 alphas emitted.

42.21. Model: The number of radioactive atoms decreases exponentially with time.

Solve: (a) ^{3}H is called tritium and has $Z=1$ and $N=2$. ^{3}H undergoes beta-minus decay and gives rise to the daughter nucleus ^{3}He, that is,

$$^3\text{H} \rightarrow {}^3\text{He} + \beta^-$$

(b) From Equation 42.18, the decay rate is

$$r = \frac{1}{\tau} = \frac{0.693}{t_{1/2}} = \frac{0.693}{12\text{ years}}\times\frac{1\text{ year}}{3.15\times10^7\text{ s}} = 1.83\times10^{-9}\text{ s}^{-1}$$

The time constant is $\tau = 5.46\times10^8$ s.

42.22. Model: The activity R of a radioactive sample is the number of decays per second.

Solve: The rate of decay is

$$R = \left|\frac{dN}{dt}\right| = rN = \frac{N}{\tau} = \frac{(0.693)N}{t_{1/2}}$$

$$\Rightarrow t_{1/2} = \frac{(0.693)N}{R} = \frac{(0.693)(5.0\times10^{15}\text{ atoms})}{5.0\times10^8\text{ Bq}} = 6.93\times10^6\text{ s}\times\frac{1\text{ day}}{86{,}400\text{ s}} = 80.2\text{ days} \approx 80\text{ days}$$

42.23. Solve: The activity of a radioactive sample is $R = rN$. From Appendix C, the half-life of ^{60}Co is $t_{1/2} = 5.27\text{ yr} = 1.663\times10^8$ s. Thus the decay rate is

$$r = \frac{1}{\tau} = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{1.663\times10^8\text{ s}} = 4.17\times10^{-9}\text{ s}^{-1}$$

We know that the sample's activity is $R = 3.50\times10^9\text{ Bq} = 3.50\times10^9$ decays/s, so the number of atoms in the sample is

$$N = \frac{R}{r} = \frac{3.50\times10^9\text{ decays/s}}{4.17\times10^9\text{ s}^{-1}} = 8.40\times10^{17}\text{ atoms}$$

The mass of the sample is

$$M = mN = (60\times1.661\times10^{-27}\text{ kg})(8.40\times10^{17}) = 8.36\times10^{-8}\text{ kg} = 83.6\ \mu\text{g}$$

Section 42.6 Nuclear Decay Mechanisms

42.24. Solve: (a) The decay is X → ^{224}Ra + ^{4}He, so X = ^{228}Th.

(b) The decay is X → $^{207}\text{Pb} + e^- + \bar{\nu}$, so X = ^{207}Tl.

(c) The decay is $^7\text{Be} + e^- \rightarrow \text{X} + \nu$, so X = ^{7}Li.

(d) The decay is X → $^{60}\text{Ni} + \gamma$, so X = ^{60}Ni.

42.25. Solve: (a) The decay is $^{230}\text{Th} \to \text{X} + {}^{4}\text{He}$, so $\text{X} = {}^{226}\text{Ra}$.

(b) The decay is $^{35}\text{S} \to \text{X} + e^{-} + \bar{\nu}$, so $\text{X} = {}^{35}\text{Cl}$.

(c) The decay is $\text{X} \to {}^{40}\text{K} + e^{+} + \nu$, so $\text{X} = {}^{40}\text{Ca}$.

(d) The decay is $^{24}\text{Na} \to {}^{24}\text{Mg} + e^{-} + \bar{\nu} \to \text{X} + \gamma$, so $\text{X} = {}^{24}\text{Mg}$.

42.26. Solve: (a) From Appendix C, the $A = 17$ isobars are ^{17}N, ^{17}O, and ^{17}F.

(b) The isotope ^{17}O is stable but rare.

(c) The daughter nucleus for both of the unstable isotopes is ^{17}O. ^{17}N decays by beta-minus decay. ^{17}F decays by electron capture.

42.27. Solve: (a) From Appendix C, the $A = 19$ isobars are ^{19}O, ^{19}F, and ^{19}Ne.

(b) The ^{19}F isotope is stable.

(c) The daughter nucleus for both of the unstable isotopes is ^{19}F. ^{19}O decays by beta-minus. ^{19}Ne decays by beta-plus.

42.28. Model: Assume the energy released goes into the alpha particle's kinetic energy so that $K_\alpha = 5.52\text{ MeV}$.

Visualize: Use Equation 42.15: $K_\alpha = (m_\text{X} - m_\text{Y} - m_\text{He})c^2 = 5.52\text{ MeV}$. We are also given $m_\text{X} + m_\text{Y} = 452\text{ u}$.

Solve:

$$(m_\text{X} - m_\text{Y} - m_\text{He})c^2 = 5.52\text{ MeV}$$

$$(m_\text{X} - m_\text{Y} - m_\text{He}) = 5.52\text{ MeV}/c^2\left(\frac{1\text{ u}}{931.49\text{ MeV}/c^2}\right) = 0.00593\text{ u}$$

$$m_\text{X} - m_\text{Y} = 0.00593\text{ u} + m_\text{He} = 0.00593\text{ u} + 4.00260\text{ u} = 4.00853\text{ u}$$

We now have a system of two equations in two unknowns which we will add together to solve for m_X.

$$\begin{cases} m_\text{X} - m_\text{Y} = 4.00853\text{ u} \\ m_\text{X} + m_\text{Y} = 452\text{ u} \end{cases}$$

$$2m_\text{X} = 456\text{ u}$$

$$m_\text{X} = 228\text{ u}$$

Looking in Appendix C for a nucleus with a mass of 228 u which decays by α decay, we find ^{228}Th.

Assess: ^{228}Th is a step in the ^{232}Th decay series. ^{238}Th has a half-life of 1.9 yr and decays by α radiation releasing 5.52 MeV of energy.

42.29. Model: The decay is $^{239}\text{Pu} \to {}^{235}\text{U} + {}^{4}\text{He}$.

Solve: Using Appendix C to find the masses, the energy released is

$$E = \Delta mc^2 = [m(^{239}\text{Pu}) - m(^{235}\text{U}) - m(^{4}\text{He})]c^2 = [239.052157\text{ u} - 235.043924\text{ u} - 4.002602\text{ u}]c^2$$

$$= (0.00563\text{ u})c^2 \times 931.49\text{ MeV}/c^2\text{ u} = 5.24\text{ MeV}$$

Assess: Essentially all of this energy goes into the alpha particle's kinetic energy.

42.30. Model: The decay is $^{24}\text{Na} \to {}^{24}\text{Mg} + e^{-} + \bar{\nu}$.

Solve: Beta-minus decay leaves the daughter atom as a positive ion. However, the mass of the ion plus the mass of the escaping electron are the mass of a neutral atom, which is what is tabulated in Appendix C. Thus the mass loss is the mass difference between the two neutral atoms. In Appendix C we find that $m(^{24}\text{Na}) = 23.990961\text{ u}$ and $m(^{24}\text{Mg}) = 23.985042\text{ u}$. The energy released in the beta-minus decay corresponds to a mass change of 0.005919 u. The energy released is

$$E = \Delta mc^2 = (0.005919\ \text{u})c^2 \times \frac{931.49\ \text{MeV}/c^2}{1\ \text{u}} = 5.51\ \text{MeV}$$

Assess: This energy is shared between the electron and the antineutrino.

42.31. Model: The decay is $^3\text{H} \rightarrow {}^3\text{He} + \text{e}^- + \overline{\nu}$.

Solve: Beta-minus decay leaves the daughter atom as a positive ion. However, the mass of the ion plus the mass of the escaping electron is the mass of a neutral atom, which is what is tabulated in Appendix C. Thus the mass loss is the mass difference between the two neutral atoms. In Appendix C we find that $m(^3\text{H}) = 3.016049\ \text{u}$, and $m(^3\text{He}) = 3.016029\ \text{u}$. The energy released in the beta-minus decay corresponds to a mass change of 0.000020 u. The energy released is

$$E = \Delta mc^2 = (0.000020\ \text{u})(931.49\ \text{MeV/u}) = 0.0186\ \text{MeV}$$

Assess: The energy E is shared between the electron and the antineutrino.

42.32. Model: The decay is $\text{n} \rightarrow \text{p}^+ + \text{e}^- + \overline{\nu}$.

Solve: Use Table 42.2. The mass lost in this decay is

$$\Delta m = m(\text{n}) - m(\text{p}^+) - m(\text{e}^-) = 939.57\ \text{MeV/c}^2 - 938.28\ \text{MeV/c}^2 - 0.51\ \text{MeV/c}^2 = 0.78\ \text{MeV/c}^2$$

The energy released is the energy equivalent of $0.78\ \text{MeV/c}^2$, or

$$E = (0.78\ \text{MeV/c}^2)(\text{c}^2) = 0.78\ \text{MeV}$$

Assess: This energy is shared between the electron and the neutrino.

Section 42.7 Biological Applications of Nuclear Physics

42.33. Model: The gray measures the energy deposited in an irradiated material.

Solve: The energy to be absorbed by the 100 g tumor is 0.20 J. Because 1 Gy is defined as 1.00 J/kg of absorbed energy, the dose given is

$$\frac{0.20\ \text{J}}{100\ \text{g}} \times \frac{1\ \text{Gy}}{1.00\ \text{J/kg}} = 2.0\ \text{Gy}$$

42.34. Model: The rad (radiation absorbed dose) measures the energy deposited in an irradiated material.

Solve: Because $1\ \text{Gy} = 1.00\ \text{J/kg}$ of absorbed energy, 1.5 Gy corresponds to a dose of 1.5 J/kg. This means a 150 g tumor absorbs an amount of energy equal to

$$(1.50\ \text{J/kg})(0.150\ \text{kg}) = 0.225\ \text{J}$$

42.35. Solve: For α-particles, the RBE = 20. The dose in Sv of the alpha radiation is 30 Gy × 20 = 600 Sv. For γ-rays, the RBE = 1. The dose in Gy of 600 Sv of γ-rays is 600 Gy.

42.36. Model: The radiation dose in units of rems is a combination of deposited energy and biological effectiveness. The RBE for beta radiation is 1.5.

Solve: 1 Gy is defined as 1.00 J/kg of absorbed energy. In the case of the 50 kg worker, the energy absorbed per kg is

$$\frac{20 \times 10^{-3}\ \text{J}}{50\ \text{kg}} = 4.0 \times 10^{-4}\ \text{J/kg}$$

This corresponds to a dose of

$$4.0 \times 10^{-4}\ \text{J/kg} \times \frac{1\ \text{Gy}}{1.00\ \text{J/kg}} = 4.0 \times 10^{-4}\ \text{Gy}$$

The dose equivalent in rems is $(4.0 \times 10^{-4}\ \text{Gy})(1.5) = 0.060\ \text{Sv} = 60\ \text{mrem}$.

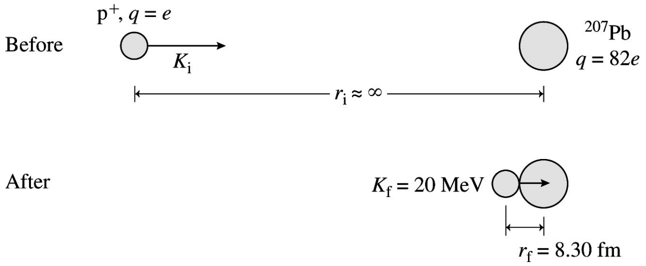
Problems

42.37. Model: Assume the ^{207}Pb nucleus remains at rest.

Visualize: The radii of the proton and the lead nucleus are

$$r_\text{p} = (1.2\text{ fm})(1)^{1/3} = 1.20\text{ fm} \qquad r_\text{Pb} = (1.2\text{ fm})(207)^{1/3} = 7.10\text{ fm}$$

At the momentum of impact on the lead nucleus, the distance between their centers is $r_\text{f} = 1.20\text{ fm} + 7.10\text{ fm} = 8.30\text{ fm}$.



Solve: The energy conservation equation $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ is

$$20\text{ MeV} + \frac{1}{4\pi\varepsilon_0}\frac{(e)(82e)}{8.30\times10^{-15}\text{ fm}} = K_\text{i} + 0\text{ J}$$

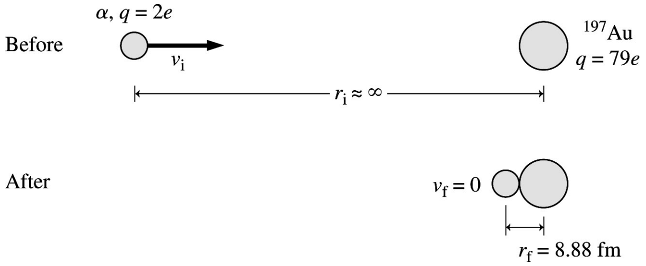
$$\Rightarrow K_\text{i} = 20\text{ MeV} + \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(82)(1.60\times10^{-19}\text{ C})^2}{8.30\times10^{-15}\text{ fm}} \times \frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}} = 34.2\text{ MeV}$$

42.38. Model: Assume the ^{197}Au nucleus remains at rest.

Visualize: The radii of the alpha particle and the gold nucleus are

$$r_\alpha = (1.2\text{ fm})(4)^{1/3} = 1.90\text{ fm} \qquad r_\text{Au} = (1.2\text{ fm})(197)^{1/3} = 6.98\text{ fm}$$

If the alpha just touches the surface of the gold nucleus, the distance between their centers is $r_\text{f} = 1.90\text{ fm} + 6.98\text{ fm} = 8.88\text{ fm}$.



Solve: (a) The energy conservation equation $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ is

$$0\text{ J} + \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{8.88\times10^{-15}\text{ fm}} = \tfrac{1}{2}mv_\text{i}^2 + 0\text{ J}$$

$$\Rightarrow v_\text{i} = \sqrt{\frac{2(9.0\times10^9\text{ N m}^2/\text{C}^2)(158)(1.60\times10^{-19}\text{ C})^2}{(4\times1.661\times10^{-27}\text{ kg})(8.88\times10^{-15}\text{ fm})}} = 3.5\times10^7\text{ m/s}$$

(b) The energy of the alpha particle is

$$K = \tfrac{1}{2}mv_\text{i}^2 = \tfrac{1}{2}(4\times1.661\times10^{-27}\text{ kg})(3.5\times10^7\text{ m/s})^2$$
$$= 4.09\times10^{-12}\text{ J}\times(1\text{ MeV}/1.60\times10^{-13}\text{ J}) = 26\text{ MeV}$$

42.39. Solve: (a) The sun's mass of 1.99×10^{30} kg is unchanged, but it assumes the density of nuclear matter, which we found to be $\rho_{\text{nuc}} = 2.3\times10^{17}\ \text{kg/m}^3$. The volume of the collapsed sun is

$$V = \frac{4}{3}\pi r^3 = \frac{M_{\text{S}}}{\rho_{\text{nuc}}} = \frac{1.99\times10^{30}\ \text{kg}}{2.3\times10^{17}\ \text{kg/m}^3} = 8.65\times10^{12}\ \text{m}^3$$

Thus its radius is

$$r = \left(\frac{3(8.65\times10^{12}\ \text{m}^3)}{4\pi}\right)^{1/3} = 12{,}700\ \text{m} = 12.7\ \text{km}$$

(b) We can use the conservation of angular momentum to find the new rotational period:

$$(I\omega)_{\text{after}} = (I\omega)_{\text{before}} \Rightarrow \omega_{\text{after}} = \frac{I_{\text{before}}}{I_{\text{after}}}\omega_{\text{before}} = \frac{\frac{2}{5}M_{\text{S}}R_{\text{S}}^2}{\frac{2}{5}M_{\text{S}}r^2}\omega_{\text{before}}$$

$$\Rightarrow \frac{2\pi}{T_{\text{after}}} = \left(\frac{R_{\text{S}}}{r}\right)^2\frac{2\pi}{T_{\text{before}}} \Rightarrow T_{\text{after}} = \left(\frac{1.27\times10^4\ \text{m}}{6.96\times10^8\ \text{m}}\right)^2(27\ \text{days}) = 7.8\times10^{-4}\ \text{s} = 780\ \mu\text{s}$$

42.40. Visualize: Let x_1 be the fraction of ^{69}Ga with atomic mass 68.92 u and x_2 be the fraction of ^{71}Ga with atomic mass 70.92 u. We know that $x_1 + x_2 = 1$ or $x_2 = 1 - x_1$. We seek x_1.

Solve:

$$x_1(68.92\ \text{u}) + x_2(70.92\ \text{u}) = 69.72$$
$$x_1(68.92\ \text{u}) + (1 - x_1)(70.92\ \text{u}) = 69.72$$
$$\Rightarrow x_1 = \frac{69.72\ \text{u} - 70.92\ \text{u}}{68.92\ \text{u} - 70.92\ \text{u}} = 0.60$$

The percent abundance of ^{69}Ga is 60%.

Assess: While Ga does not appear in Appendix C, this result can be verified in other tables of isotopes of the elements.

42.41. Solve: (a) The binding energy of the electron in a hydrogen atom is $B = 13.6$ eV. That is, the mass decreases by the equivalent of 13.6 eV when an electron and proton form a hydrogen atom. Since $B = \Delta mc^2$,

$$\Delta m = \frac{13.6\ \text{eV}}{c^2} = \frac{13.6\ \text{eV}}{c^2}\times\frac{1\ \text{u}}{931.49\ \text{MeV}/c^2} = 1.46\times10^{-8}\ \text{u}$$

As a percentage of the hydrogen mass, the mass decrease is

$$\frac{\Delta m}{1.007825\ \text{u}} = \frac{1.46\times10^{-8}\ \text{u}}{1.007825\ \text{u}} = 1.45\times10^{-6}\%$$

(b) The mass decrease is $\Delta m = 2m_{\text{p}} + 2m_{\text{n}} - m_{\text{He nuc}}$. These are nuclear masses, but Appendix C tabulates atomic masses. Add and subtract the mass of two electrons:

$$\Delta m = 2(m_{\text{p}} + m_{\text{e}}) + 2m_{\text{n}} - (m_{\text{He nuc}} - 2m_{\text{e}}) = 2m_{\text{H}} + 2m_{\text{n}} - m_{\text{He}}$$

where m_{H} is the mass of a hydrogen atom and m_{He} is the mass of a helium atom. Using Appendix C,

$$\Delta m = 2(1.007825\ \text{u}) + 2(1.008665) - 4.002602 = 0.0304\ \text{u}$$

As a percentage of the helium mass, the mass decrease is

$$\frac{0.0304\ \text{u}}{4.0026\ \text{u}} = 0.0076 = 0.76\%$$

(c) Although mass does change in chemical reactions, the change is an incredibly small fraction of the mass of the atoms. No experiment will be sensitive to changes of $\approx 1\times 10^{-6}\%$, so this small change in mass is easily neglected. Not so in nuclear reactions, where the mass change can be ≈1% of the particle masses. Not only is this mass change easily detectable, it is essential for understanding nuclear reactions.

42.42. Visualize: Please refer to Figure 42.6 for the graph of the binding energy versus mass number.
Solve: For a ^{4}He nucleus, the binding energy per nucleon is 7.2 MeV. Because $A = 4$, the binding energy of the ^{4}He nuclei is $7.2\text{ MeV}\times 4 = 28.8$ MeV. Three ^{4}He nuclei thus have a total energy of $28.8\text{ MeV}\times 3 = 86.4$ MeV. For the ^{12}C nucleus, the binding energy per nucleon is 7.7 MeV. Because $A = 12$, the binding energy of ^{12}C is $7.7\text{ MeV}\times 12 = 92.4$ MeV. When three ^{4}He nuclei fuse together to form a ^{12}C nucleus, the total energy released is 92.4 MeV − 86.4 MeV = 6.0 MeV.

42.43. Visualize: Please refer to Figure 42.6 for the graph of the binding energy versus mass number.
Solve: For a nucleus of mass number of 240, the binding energy per nucleon is ≈7.8 MeV. Hence, the total binding is $(7.8\text{ MeV})(240) = 1870$ MeV. For a nucleus of mass number of 120, the binding energy per nucleon is ≈8.5 MeV and the total binding energy is $(8.5\text{ MeV})(120) = 1020$ MeV. When a nucleus with mass number 240 fissions into two nuclei with mass number 120, the total energy released is

$$2(1020)\text{ MeV} - 1870\text{ MeV} = 170\text{ MeV}$$

42.44. Visualize: Please refer to Figure 42.6 for the graph of the binding energy versus mass number.
Solve: For a ^{56}Fe nucleus, the binding energy per nucleon is 8.8 MeV. Because $A = 56$, the total binding energy is $8.8\text{ MeV}\times 56 = 493\text{ MeV}$. For a ^{28}Al nucleus, the binding energy per nucleon is 8.4 MeV and the total binding energy is $28\times 8.4\text{ MeV} = 235\text{ MeV}$. The total binding energy for two ^{28}Al nuclei is $2\times 235\text{ MeV} = 470\text{ MeV}$. Because the ^{56}Fe is more tightly bound than two ^{28}Al nuclei, ^{56}Fe cannot fission spontaneously. It could perhaps be forced to fission with the input of 493 MeV − 470 MeV = 23 MeV.

42.45. Solve: The radius of a ^{238}U nucleus is

$$r = r_0 A^{1/3} = (1.2\text{ fm})(238)^{1/3} = 7.436\text{ fm}$$

For the de Broglie wavelength to be equal to the diameter of the ^{238}U nucleus, $p = h/\lambda = h/2r$. Hence,

$$K = \frac{p^2}{2m} = \left(\frac{h}{2r}\right)^2 \frac{1}{2m} = \left(\frac{6.63\times 10^{-34}\text{ J s}}{2\times 7.436\times 10^{-15}\text{ m}}\right)^2 \frac{1}{2(4\times 1.67\times 10^{-27}\text{ kg})} \times \frac{1\text{ eV}}{1.6\times 10^{-19}\text{ J}} = 0.93\text{ MeV}$$

42.46. Solve: From Appendix C, the half-life of a ^{133}Cs sample is

$$t_{1/2} = 30\text{ years} \times \frac{3.15\times 10^7\text{ s}}{1\text{ year}} = 9.45\times 10^8\text{ s}$$

The activity of a radioactive sample is $R = rN$. The decay rate is

$$r = \frac{1}{\tau} = \frac{0.693}{t_{1/2}} = \frac{0.693}{9.45\times 10^8\text{ s}} = 7.33\times 10^{-10}\text{ s}^{-1}$$

$$\Rightarrow N = \frac{R_0}{r} = \frac{2.0\times 10^8\text{ Bq}}{7.33\times 10^{-10}\text{ s}^{-1}} = 2.7\times 10^{17}\text{ atoms}$$

That is, at $t = 0$ s, we have 2.7×10^{17} ^{137}Cs atoms and after a very long time all will have decayed. Thus 2.7×10^{17} beta particles will have been emitted.

42.47. Model: The ^{14}C/^{12}C ratio in living matter is 1.3×10^{-12}. After death, this ratio decays exponentially with time.
Solve: The decay equation in terms of N, the number of radioactive atoms, can equally well be written in terms of the ratio $R = N/N_{\text{ref}}$ where N_{ref} is a stable reference. In dating experiments, the number of ^{12}C atoms is a reference to which the number of ^{14}C atoms can be compared. Thus

$$R = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = (1.3\times10^{-12})\left(\frac{1}{2}\right)^{t/5730\text{ yr}}$$

where $R_0 = 1.3\times10^{-12}$ is the ratio at the time of death and the half-life of ^{14}C is known to be 5730 yr. We can find the time at which the ratio has decreased to R by taking the logarithm of both sides:

$$\left(\frac{1}{2}\right)^{t/5730\text{ yr}} = \frac{R}{1.3\times10^{-12}} \Rightarrow \frac{t}{5730\text{ yr}}\ln\left(\frac{1}{2}\right) = \ln\left(\frac{R}{1.3\times10^{-12}}\right) \Rightarrow t = \frac{\ln(1.3\times10^{-12}/R)}{\ln(2)}\cdot 5730\text{ yr}$$

This sample has $R = 1.65\times10^{-13}$, so its age is

$$t = \frac{\ln(1.3\times10^{-12}/1.65\times10^{-13})}{\ln(2)}\cdot 5730\text{ yr} = 17{,}100\text{ yr}$$

42.48. Visualize: Appendix C gives the half-life for ^{238}U: $t_{1/2} = 4.47\times10^{9}\text{ yr} = 1.41\times10^{17}\text{ s}$. We are given $R_0 = 1\text{ Ci} = 3.7\times10^{10}\text{ Bq}$.

Solve: Use $r = 1/\tau$ to compute the number of uranium atoms.

$$N_0 = \frac{R_0}{r} = R_0\frac{t_{1/2}}{\ln 2} = (3.7\times10^{10}\text{ Bq})\frac{1.41\times10^{17}\text{ s}}{\ln 2} = 7.53\times10^{27}$$

Now multiply this by the mass of one uranium atom to get the total mass.

$$M = N_0 m_{\text{U}} = (7.53\times10^{27})(238.1\text{ u})(1.661\times10^{-27}\text{ kg/u}) = 3000\text{ kg}$$

Assess: This large mass shows that 1 Ci is indeed a significant radiation source.

42.49. Visualize: Appendix C gives the half-life for ^{137}Cs: $t_{1/2} = 30\text{ yr} = 9.467\times10^{8}\text{ s}$. We are given $R_0 = 550\text{ mCi} = (550\times10^{-3})(3.7\times10^{10}\text{ Bq}) = 2.035\times10^{10}\text{ Bq}$.

Solve: **(a)** Use $r = 1/\tau$ to compute the number of cesium atoms.

$$N_0 = \frac{R_0}{r} = R_0\frac{t_{1/2}}{\ln 2} = (2.035\times10^{10}\text{ Bq})\frac{9.467\times10^{8}\text{ s}}{\ln 2} = 2.78\times10^{19}$$

Now multiply this by the mass of one cesium atom to get the total mass.

$$M = N_0 m_{\text{Cs}} = (2.78\times10^{19})(136.9\text{ u})(1.661\times10^{-27}\text{ kg/u}) = 6.12\times10^{-6}\text{ kg}$$

(b) Solve for t in the equation for activity $R = R_0 e^{-t/\tau}$.

$$t = -\tau\ln\left(\frac{R}{R_0}\right) = -\frac{t_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right) = -\frac{9.467\times10^{8}\text{ s}}{\ln 2}\ln\left(\frac{25\text{ mCi}}{550\text{ mCi}}\right) = 4.22\times10^{9}\text{ s} = 130\text{ yr}$$

Assess: This answer shows how important proper cleanup of radioactive spills is.

42.50. Model: The number of radioactive tracers decays exponentially.

Solve: **(a)** The activity of a radioactive sample decays as

$$R = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \Rightarrow 0.95\text{ mCi} = 1.15\text{ mCi}\left(\frac{1}{2}\right)^{16\text{ hours}/t_{1/2}} \Rightarrow \ln\left(\frac{95}{115}\right) = \left(\frac{16\text{ hours}}{t_{1/2}}\right)\ln\left(\frac{1}{2}\right) \Rightarrow t_{1/2} = 58.0\text{ hours}$$

(b) Let t be the time it takes the activity to drop from 0.95 mCi to 0.10 mCi. Using the results of part (a),

$$0.10\text{ mCi} = 0.95\text{ mCi}\left(\frac{1}{2}\right)^{t/58.0\text{ hours}} \Rightarrow \ln\left(\frac{10}{95}\right) = \left(\frac{t}{58.0\text{ hours}}\right)\ln\left(\tfrac{1}{2}\right) \Rightarrow t = 188\text{ hours}$$

42.51. Model: The decay is $^{223}\text{Ra} \rightarrow {}^{219}\text{Rn} + {}^{4}\text{He}$.

Solve: (a) The energy released in the above decay is

$$E = [m(^{223}\text{Ra}) - m(^{219}\text{Rn}) - m(^{4}\text{He})] \times 931.49\ \text{MeV/u}$$
$$= [223.018499\ \text{u} - 219.009477\ \text{u} - 4.002602\ \text{u}] \times 931.49\ \text{MeV/u} = 5.98\ \text{MeV}$$

That is, each α-particle is released with an energy of

$$5.98\ \text{MeV} = 5.98 \times 10^6\ \text{eV} \times 1.6 \times 10^{-19}\ \text{J/eV} = 9.57 \times 10^{-13}\ \text{J}$$

The amount of energy needed to raise the temperature of 100 mL of water at 18°C to 100°C is

$$Q = mc\Delta t = (0.10\ \text{L})\left(\frac{1\ \text{kg}}{1\ \text{L}}\right)(4190\ \text{J/kg K})(100\ \text{K} - 18\ \text{K}) = 34{,}400\ \text{J}$$

The number of decays we need to generate this amount of energy is

$$\frac{34{,}400\ \text{J}}{9.57 \times 10^{-13}\ \text{J}} = 3.59 \times 10^{16}$$

The total number of radium atoms in the cube at $t = 0$ s is

$$N_0 = \frac{1\ \text{g}}{223\ \text{g/mol}} \times 6.02 \times 10^{23}\ \text{atoms/mol} = 2.70 \times 10^{21}\ \text{atoms}$$

The number of needed decays is very small compared to N_0, so we can write

$$\frac{dN}{dt} \approx \frac{\Delta N}{\Delta t} = -rN \quad \Rightarrow \quad \Delta t = -\frac{\Delta N}{rN} = -\tau\frac{\Delta N}{N} = -\frac{t_{1/2}}{\ln 2}\frac{\Delta N}{N}$$

where we used $r = 1/\tau$. $\Delta N = -3.59 \times 10^{16}$, with the negative sign due to the fact that the number of radium atoms decreases. Since $t_{1/2} = 11.43\ \text{days} = 9.876 \times 10^5\ \text{s}$, the time for 3.59×10^{16} decays is

$$\Delta t = -\frac{9.876 \times 10^5\ \text{s}}{\ln 2}\frac{(-3.59 \times 10^{16})}{2.70 \times 10^{21}} = 18.9\ \text{s} \approx 19\ \text{s}$$

(b) It's possible that an alpha-particle collision will break the molecular bond in a *very* small number of H_2O molecules, causing H_2 gas and O_2 gas to bubble out of the water. The water that remains in the container has not changed or been altered.

42.52. Model: The number of radioactive atoms decreases exponentially with time.

Solve: (a) If 90% of the sample has decayed, 10% is still present. This happens at time t such that

$$N = 0.10N_0 = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \Rightarrow \left(\frac{1}{2}\right)^{t/t_{1/2}} = 0.10 \Rightarrow \frac{t}{t_{1/2}}\ln\left(\frac{1}{2}\right) = \ln(0.10)$$

$$t = \frac{\ln(0.10)}{\ln(0.50)}t_{1/2} = 3.32t_{1/2}$$

That is, 90% decay in 3.32 half-lives.

(b) When 99% of the sample has decayed, 1% is left. The analysis is the same as in part (a), giving

$$t = \frac{\ln(0.010)}{\ln(0.50)}t_{1/2} = 6.64t_{1/2}$$

That is, 99% decay in 6.64 half-lives.

42.53. Model: The number of radioactive atoms decays exponentially with time.

Solve: At $t = 2$ hours, the number of A and B atoms is equal. Thus

$$N_A = (N_A)_0 \left(\frac{1}{2}\right)^{(2.0\text{ h})/(0.5\text{ h})} = \frac{(N_A)_0}{16} = N_B = (N_B)_0 \left(\frac{1}{2}\right)^{(2.0\text{ h})/t_{1/2}}$$

Initially, $(N_A)_0 = 5(N_B)_0$. With this information,

$$\frac{5}{16} = \left(\frac{1}{2}\right)^{(2.0\text{ h})/t_{1/2}} \Rightarrow \ln(5/16) = \frac{2.0\text{ h}}{t_{1/2}} \ln(1/2) \Rightarrow t_{1/2} = \frac{\ln(1/2)}{\ln(5/16)} \cdot 2\text{ h} = 1.19\text{ h} \approx 1.2\text{ h}$$

42.54. Model: The number of ^{131}Ba and ^{47}Ca atoms decays exponentially.

Solve: From Equation 42.8, the number of radioactive atoms is $N = N_0 e^{-t/\tau}$. At time $t = 0$ s, $(N_0)_{\text{Ca}} = 2(N_0)_{\text{Ba}}$. The ratio of atoms at time t is

$$\frac{N_{\text{Ca}}}{N_{\text{Ba}}} = \frac{(N_0)_{\text{Ca}}}{(N_0)_{\text{Ba}}} \frac{e^{-t/\tau_{\text{Ca}}}}{e^{-t/\tau_{\text{Ba}}}}$$

Relating the half-life to the time constant,

$$\tau_{\text{Ca}} = \frac{(t_{1/2})_{\text{Ca}}}{\ln 2} = \frac{4.5\text{ days}}{\ln 2} = 6.492\text{ days} \qquad \tau_{\text{Ba}} = \frac{(t_{1/2})_{\text{Ba}}}{\ln 2} = \frac{12\text{ days}}{\ln 2} = 17.31\text{ days}$$

$$\Rightarrow \frac{N_{\text{Ca}}}{N_{\text{Ba}}} = (2)\frac{e^{-(2.5)7\text{ days}/6.492\text{ days}}}{e^{-(2.5)7\text{ days}/17.31\text{ days}}} = \frac{2(0.0675)}{0.3639} = 0.371$$

42.55. Model: The number of ^{235}U atoms decays exponentially with time.

Solve: The ^{235}U decays as

$$N_U = (N_U)_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

The ratio of ^{235}U atoms when the earth formed to the number now present is

$$\frac{(N_U)_0}{N_U} = \frac{1}{(1/2)^{t/t_{1/2}}} = (2)^{t/t_{1/2}} = (2)^{(4500\text{ my})/(700\text{ my})} = 86.1 \approx 86$$

That is, the abundance of ^{235}U was 86 times larger then than it is now.

42.56. Solve: Since the dose equivalent in Sv is the dose in Gy times the RBE, the dose is

$$\frac{0.30\times10^{-3}\text{ Sv}}{0.85} = 3.53\times10^{-4}\text{ Gy}\times\frac{1.00\text{ J/kg}}{1\text{ Gy}} = 3.53\times10^{-4}\text{ J/kg}$$

Since only one-fourth of the body received x-ray exposure, the total amount of energy received is

$$3.53\times10^{-4}\text{ J/kg}\times\frac{1}{4}\times60\text{ kg} = 5.29\times10^{-3}\text{ J}$$

The energy of each photon is

$$10\text{ keV}\times\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}} = 1.6\times10^{-15}\text{ J}$$

Thus, the number of x-ray photons absorbed by the body is

$$\frac{5.29\times10^{-3}\text{ J}}{1.6\times10^{-15}\text{ J}} = 3.3\times10^{12}$$

42.57. Model: The rate of removal of a tracer is the rate of decay plus the rate of excretion.
Solve: The rate at which the number of tracer atoms in the body decays is

$$\frac{dN}{dt}=\left(\frac{dN}{dt}\right)_{\text{decay}}+\left(\frac{dN}{dt}\right)_{\text{excrete}}=-r_{\text{d}}N-r_{\text{e}}N=-(r_{\text{d}}+r_{\text{e}})N=-r_{\text{eff}}N$$

where r_{d} and r_{e} are the decay rate and the excretion rate. The effective loss rate is the sum of these two rates. The effective half-life is

$$t_{1/2}=\frac{\ln 2}{r_{\text{eff}}}=\frac{\ln 2}{r_{\text{d}}+r_{\text{e}}}=\frac{\ln 2}{\ln 2/(t_{1/2})_{\text{d}}+\ln 2/(t_{1/2})_{\text{e}}}=\left[\frac{1}{(t_{1/2})_{\text{d}}}+\frac{1}{(t_{1/2})_{\text{e}}}\right]^{-1}$$

The effective half-life is the inverse of the sum of the inverses of the half-lives of the individual processes. Note that this is exactly like finding the equivalent resistance of two parallel resistors. Here the two removal processes run "in parallel." Using the known values,

$$t_{1/2}=\left[\frac{1}{9.0\text{ days}}+\frac{1}{6.0\text{ days}}\right]^{-1}=3.6\text{ days}$$

42.58. Model: The number of ^{239}Pu atoms decays exponentially.

Solve: (a) The number of ^{239}Pu atoms in a 1.0-μm-diameter particle is

$$N=\left(\frac{m}{M_A}\right)N_A=\frac{(\rho V)}{M_A}N_A=\frac{\rho\left(\frac{4\pi}{3}\right)r^3N_A}{M_A}$$

$$=\frac{(19.8\text{ kg/m}^3)\left(\frac{4\pi}{3}\right)(0.5\times10^{-6}\text{ m})^3(6.02\times10^{23}\text{ mol}^{-1})}{239\times10^{-3}\text{ kg/mol}}=2.61\times10^7\approx2.6\times10^7$$

(b) The activity of the particle is

$$R=\left|\frac{dN}{dt}\right|=rN=\frac{\ln 2}{t_{1/2}}N=\frac{0.6931}{24{,}000\text{ years}\times3.15\times10^7\text{ s/year}}\times2.61\times10^7=2.39\times10^{-5}\text{ Bq}\approx2.4\times10^{-5}$$

(c) The volume of the 50-μm diameter sphere of tissue around the particle is

$$\frac{4\pi}{3}(25\times10^{-6}\text{ m})^3=6.545\times10^{-14}\text{ m}^3$$

This volume of the tissue has a mass of

$$m=\rho V=(1000\text{ kg/m}^3)(6.545\times10^{-14}\text{ m}^3)=6.545\times10^{-11}\text{ kg}$$

In one year, the activity changes insignificantly. Thus the number of decays per year is

$$\left|\frac{dN}{dt}\right|\Delta t=(2.39\times10^{-5}\text{ Bq})(3.15\times10^7\text{ s})=7.54$$

Since each decay creates an α-particle with energy 5.2 MeV, the total energy received per year by the tissue is

$$(7.529\times10^5\times5.2\text{ MeV})\times(1.6\times10^{-19}\text{ J/eV})=6.264\times10^{-10}\text{ J}$$

Dividing this energy by the tissue's mass, the dose received by the tissue is

$$6.264\times10^{-10}\text{ J}/6.545\times10^{-11}\text{ kg}=9.586\text{ J/kg}=9.586\text{ Gy}$$

The dose per year in mSv is

$$(9.586\text{ Gy})(\text{RBE})=(9.586\text{ Gy})(20)=191.7\text{ Sv}\approx1.9\times10^5\text{ mSv}$$

(d) This is a very high dose to a very small volume of body mass. Table 42.5 gives a typical exposure (in units of mrem/year) from various radiation sources. The background radiation from various natural-occurring sources is about 3 mSv/year. The exposure to this tissue is much higher than the background level.

42.59. Model: Although individual radon atoms decay, the radon is always being replaced such that the radon concentration remains constant with a constant activity of 4 pCi/L.

Solve: (a) Activity is *R* = *rN*, so the number of atoms giving rise to a known activity is *N* = *R*/*r*. The half-life of ^{222}Rn is 3.82 days = 330,000 s, so its decay rate is

$$r = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{330{,}000} = 2.1\times10^{-6}\ \text{s}^{-1}$$

One liter is $1\ \text{L} = 1000\ \text{cm}^3 = 0.0010\ \text{m}^3$, so $1\ \text{m}^3$ of air is 1000 L. At 4 pCi/L, the activity in $1\ \text{m}^3$ is

$$R = 4000\ \text{pCi} = 4\times10^{-9}\ \text{Ci}\times\frac{3.7\times10^{10}\ \text{Bq}}{1\ \text{Ci}} = 148\ \text{Bq}$$

Thus the number of ^{222}Rn atoms in $1\ \text{m}^3$ of air is

$$N = \frac{R}{r} = \frac{148\ \text{Bq}}{2.10\times10^{-6}\ \text{s}^{-1}} = 7.05\times10^{7}\ \text{atoms} \approx 7\times10^{7}\ \text{atoms}$$

(b) We first need to find how many ^{222}Rn atoms decay within 3 cm of the person in 1 year. The person is a 25-cm-diameter cylinder 180 cm tall. The decays must be inside a 31-cm-diameter cylinder (i.e., 25 cm + 3 cm + 3 cm) 183 cm tall. The volume of air in this shell is

$$V = \pi(0.155\ \text{m})^2(1.83\ \text{m}) - \pi(0.125\ \text{m})^2(1.80\ \text{m}) = 0.0498\ \text{m}^3$$

The concentration of ^{222}Rn is $7.05\times10^{7}\ \text{m}^{-3}$, so the number of ^{222}Rn atoms in this shell is $N = (0.0498\ \text{m}^3)\times(7.05\times10^{7}\ \text{m}^{-3}) = 3.51\times10^{6}$. The total number of decays per second is

$$R = rN = (2.10\times10^{-6}\ \text{s}^{-1})(3.51\times10^{6}) = 7.4\ \text{decays per second}$$

Only half of these decays direct the alpha particle toward the person, so the number of alphas striking the person in 1 year is

$$N_\alpha = \frac{1}{2}\times(7.4\ \text{decays/s})\times(3.15\times10^{7}\ \text{s/yr}) = 1.2\times10^{8}\ \text{alphas/yr}$$

Each alpha deposits $5.50\ \text{MeV} = 8.8\times10^{-13}\ \text{J}$ of energy, so the energy absorbed in 1 year is $E = (1.2\times10^{8}\ \text{alphas})\times(8.8\times10^{-13}\ \text{J/alpha}) = 1.06\times10^{-4}\ \text{J}$. The dose of alpha radiation (with RBE = 20) received by a 65 kg person in a year is

$$\text{dose} = \frac{1.06\times10^{-4}\ \text{J}}{65\ \text{kg}}\times\frac{1\ \text{Gy}}{1.00\ \text{J/kg}}\times20 = 3.3\times10^{-2}\ \text{mSv}$$

(c) This dose is roughly 1% of the natural background of radioactivity. Although an exposure increase of 1% may seem too small to worry about, we assumed in using a 65 kg mass that the radiation is uniformly received. But alpha radiation is not very penetrating, so the skin and outer extremities would receive a dose considerably higher than 2.4 mrem. Since some people are housebound and really would receive this yearly dose, an upper limit of 4 pCi/L seems prudent.

Challenge Problems

42.60. Model: The number of ^{40}K atoms decays exponentially with time.

Solve: Suppose the number of ^{40}K atoms is N_0 at the time the lava solidifies. There are no ^{40}Ar atoms at that time. As time passes, 11% of the ^{40}K that decays goes into ^{40}Ar, but the total number of atoms locked inside the lava is unchanged. That is, $N_{\text{Ar}} = (0.11)(N_0, -N_{\text{K}})$ where N_{K} is the remaining number of ^{40}K atoms. Dividing by N_{K}, we have

$$\frac{N_{\text{Ar}}}{N_{\text{K}}} = (0.11)\left(\frac{N_0}{N_{\text{K}}} - 1\right) \Rightarrow \frac{0.013}{0.11} + 1 = \frac{N_0}{N_{\text{K}}} \Rightarrow \frac{N_{\text{K}}}{N_0} = \frac{1}{1.118} = 0.894$$

That is, 89.4% of the ^{40}K remains at a time when the Ar/K ratio is 0.013. The ^{40}K decays as

$$N_{\text{K}} = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \Rightarrow \frac{t}{t_{1/2}}\ln(1/2) = \ln(N_{\text{K}}/N_0)$$

$$\Rightarrow t = (1.28 \text{ billion years})\frac{\ln(0.894)}{\ln(0.50)} = 0.21 \text{ billion years} = 210 \text{ million years}$$

42.61. Model: The decay is $^{A}\text{X}_Z \rightarrow {}^{A}\text{Y}_{Z-1} + \text{e}^+ + \nu$.

Solve: (a) This decay is energetically possible only if the mass of the X nucleus exceeds the mass of the Y nucleus plus the mass of the positron. The X nucleus contains Z protons and N neutrons while the Y nucleus has $(Z-1)$ protons and $(N+1)$ neutrons. Thus the mass requirement is

$$Zm_{\text{p}} + Nm_{\text{n}} > (Z-1)m_{\text{p}} + (N+1)m_{\text{n}} + m_{\text{e}}$$

Tabulated masses are atomic masses, which include the electron masses. Atom X has Z electrons and atom Y has $(Z-1)$ electrons. Add the term Zm_{e} to both sides, obtaining

$$Zm_{\text{p}} + Zm_{\text{e}} + Nm_{\text{n}} > (Z-1)m_{\text{p}} + Zm_{\text{e}} + (N+1)m_{\text{n}} + m_{\text{e}}$$

$$\Rightarrow Z(m_{\text{p}} + m_{\text{e}}) + Nm_{\text{n}} > [(Z-1)(m_{\text{p}} + m_{\text{e}}) + (N+1)m_{\text{n}}] + 2m_{\text{e}}$$

The term on the left is the mass of an $^{A}\text{X}_Z$ atom and the term in square brackets on the right is the mass of an $^{A}\text{Y}_{z-1}$ atom. Thus the threshold condition for beta-plus decay is

$$m(^{A}\text{X}_z) > m(^{A}\text{Y}_{z-1}) + 2m_{\text{e}}$$

(b) From Appendix C, $m(^{13}\text{N}) = 13.005738$ u and $m(^{13}\text{C}) = 13.003355$ u. Thus $m(^{13}\text{C}) + 2m_{\text{e}} = 13.004455$ u. This is less than $m(^{13}\text{N})$, so beta-plus decay is allowed. The energy released is

$$E = \Delta mc^2 = (0.001283 \text{ u})(931.49 \text{ MeV/u}) = 1.20 \text{ MeV}$$

42.62. Model: The number of ^{235}U and ^{238}U atoms decays exponentially with time.

Solve: From Appendix C, the half-life of ^{235}U is $(t_{1/2})_{235} = 7.04\times10^8$ yr and that of ^{238}U is $(t_{1/2})_{238} = 4.47\times10^9$ yr. Today, $N_{238}/N_{235} = 0.9928/0.0072 = 137.9$, but it is thought that $(N_{238})_0/(N_{235})_0 \approx 1$ immediately after the supernova that created these elements. The decays are such that

$$\frac{N_{238}}{N_{235}} = \frac{(N_{238})_0(1/2)^{t/(t_{1/2})_{238}}}{(N_{235})_0(1/2)^{t/(t_{1/2})_{235}}} \Rightarrow \frac{(1/2)^{t/(t_{1/2})_{238}}}{(1/2)^{t/(t_{1/2})_{235}}} = \left(\frac{1}{2}\right)^{t/(t_{1/2})_{238} - t/(t_{1/2})_{235}} = 137.9$$

To solve for t, take the logarithm of both sides:

$$\left[\left(\frac{t}{t_{1/2}}\right)_{238} - \left(\frac{t}{t_{1/2}}\right)_{235}\right]\ln(1/2) = t\times\left[\left(\frac{1}{t_{1/2}}\right)_{238} - \left(\frac{1}{t_{1/2}}\right)_{235}\right]\ln(1/2) = \ln 137.9$$

$$\Rightarrow t = \frac{\ln 137.9}{\left[(1/t_{1/2})_{238} - (1/t_{1/2})_{235}\right]\ln(1/2)} = 5.9\times10^9 \text{ years} \approx 6 \text{ billion years}$$

The supernova occurred approximately 6 billion years ago.

Assess: Our sun and solar system are ≈4.5 billion years old, so the supernova occurred ≈1.5 billion years before the debris coalesced to form the solar system.

42.63. Model: A quantum particle can tunnel through a classically forbidden region.

Solve: (a) Kinetic energy is $K = E - U$. The alpha particle has $E = 5$ MeV whether it is inside or outside the nucleus. Inside, where $U = -60$ MeV, the kinetic energy is $K_{\text{in}} = 65.0$ MeV. Outside, where $U = 0$ MeV, the kinetic energy is $K_{\text{out}} = 5.0$ MeV.

(b) The particle's speed is found from

$$\tfrac{1}{2}mv^2 = K_{\text{in}} \Rightarrow v = \sqrt{\frac{2(65\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{4\times(1.661\times10^{-27}\ \text{kg})}} = 5.60\times10^7\ \text{m/s}$$

(Although this is $>0.1c$, the spirit of this simple model of alpha decay is best served with a classical calculation rather than a more complex relativistic calculation of the speed.) The time needed to move from one side of the potential well to the other, a distance of 15 fm, is

$$\Delta t = \frac{15\times10^{-15}\ \text{m}}{5.60\times10^7\ \text{m/s}} = 2.7\times10^{-22}\ \text{s}$$

The particle collides with one wall or the other of the potential-energy barrier each time it moves across the potential well, so the rate of collisions is

$$R = \frac{1}{\Delta t} = \frac{1}{2.7\times10^{-22}\ \text{s per collision}} = 3.7\times10^{21}\ \text{collisions/s}$$

(c) From Chapter 41, the tunneling probability is $P_{\text{tunnel}} = e^{-2w/\eta}$, where w is the width of the barrier and η is the penetration distance into the classically forbidden region:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{1.05\times10^{-34}\ \text{J s}}{\sqrt{2(4\times1.661\times10^{-27}\ \text{kg})(25\times10^6\ \text{eV}\times1.60\times10^{-19}\ \text{J/eV})}}$$
$$= 4.55\times10^{-16}\ \text{m} = 0.455\ \text{fm}$$

The probability of tunneling through the 20-fm-wide barrier is

$$P_{\text{tunnel}} = e^{-2(20\ \text{fm})/(0.455\ \text{fm})} = 6.6\times10^{-39}$$

(d) Although the probability of tunneling is extremely small, the alpha particle collides with the barrier a very large number of times per second. The probability that the particle is still inside the nucleus after N collisions is $P_{\text{in}} \approx 1 - NP_{\text{tunnel}}$. The number of collisions needed to reduce P_{in} to 0.50, at which time half of a sample of nuclei would have decayed, is

$$N = \frac{1 - P_{\text{in}}}{P_{\text{tunnel}}} = \frac{0.50}{6.6\times10^{-39}} = 7.6\times10^{37}\ \text{collisions}$$

At a collision rate of 3.7×10^{21} collisions/s, we can estimate the half-live to be

$$t_{1/2} = \frac{7.6\times10^{37}\ \text{collisions}}{3.7\times10^{21}\ \text{collisions/s}} = 2.05\times10^{16}\ \text{s}\times\frac{1\ \text{year}}{3.15\times10^7\ \text{s}}$$
$$= 6.5\times10^8\ \text{years} = 650\ \text{million years}$$

Assess: The tunneling probability is *very* sensitive to the value of η. If you round your results slightly differently and get a slightly different value of η, your answer for $t_{1/2}$ could differ by more than 100 million years. Even so, heavy nuclei that decay by the emission of alpha particles with $K \approx 5$ MeV usually do have $t_{1/2}$ of several hundred million years.